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$$+(a^2d-h^2b-beu)^4\Big\}du$$

$$=\frac{1}{60a^2b^3c^2h}\Big\{(a^2+bd)^5-(a^2-be)^5-b^5e^5-b^5d^5\Big\}$$

$$=\frac{1}{120\triangle a^2b^2c^2}\Big\{\Big(\frac{a^2+b^2+c^2}{2}\Big)^5-\Big(\frac{a^2+c^2-b^2}{2}\Big)^5-\Big(\frac{a^2+b^2-c^2}{2}\Big)^5-\Big(\frac{b^2+c^2-a^2}{2}\Big)^5\Big\}$$

$$=\frac{4\triangle^4}{15a^2b^2c^2}\Big\{(\cot A+\cot B+\cot C)^5-\cot^5 A-\cot^5 B-\cot^5 C\Big\}$$

$$=\frac{a^4+b^4+c^4}{48\triangle}.$$

The last four expressions are the same and show the beautiful relations existing between the terms of the triangle.

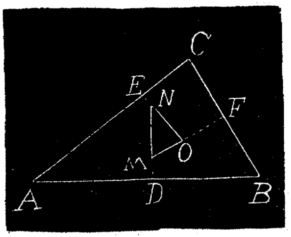
This problem was also solved by *F. P. MATZ*. His solution is published in this issue as solution of problem 24, that problem being identical with 23

24. Proposed by F P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

The average area of the triangle formed by three perpendiculars drawn from the sides of the triangle (a, b, c), is $\mathbf{A}=(a^4+b^4+c^4)/48\triangle$.

Solution by the PROPOSER.

Let $BC=a$, $CA=b$, $AB=c$, $BF=x$, $CE=y$, and $AD=z$. The $\triangle MNO$ is similar to the $\triangle ABC$; and it may be *wholly within*, *partly within*, or *wholly without*, the $\triangle ABC$. Put $\angle NAD=\phi$, and $\angle MBD=\theta$; then $AN=(b-y)/\cos(A-\phi)$ $=z\cos\phi\ldots.(\alpha)$, and $ND=z\tan\phi\ldots.(\beta)$. By means of (α), we have from (β), $ND=[(b-y)-z\cos A]/\sin A\ldots.(1)$. Also, $BM=(c-z)/\cos(B-\theta)=x/\cos\theta\ldots.(\gamma)$, and $MD=(c-z)\tan(B-\theta)\ldots.(\delta)$. By means of (γ) we have from (δ), $MD=[x-(c-z)\cos B]/\sin B\ldots.(2)$. Subtracting (2) from (1), we have the expression:



$$MN=\frac{b\sin B+c\sin A\cos B-(x\sin A+y\sin B+z\sin C)}{\sin A\sin B}\ldots.(3).$$

The *first two* terms in the numerator of (3) may be symmetrically written thus: $a\sin B\cos C+b\sin C\cos A+c\sin A\cos B,=\frac{1}{2}(a\sin A+b\sin B+c\sin C)\ldots.(\varepsilon)$. Transforming (3) by means of (ε), we have

$$MN=\frac{(a-2x)\sin A+(b-2y)\sin B+(c-2z)\sin C}{2\sin A\sin B}\ldots.(4).$$

Representing the numerator of the right-hand member of (4) by $\mathbf{N}$, we have (by symmetry) the expressions:

$$NO=\mathbf{N}/2\sin C\sin A,\text{ and }OM=\mathbf{N}/2\sin B\sin C;$$

and, consequently, the $\triangle MNO=\frac{1}{8}(\mathbf{N})^2 / \sin A \sin B \sin C \ldots\ldots(5)$. The expression for the average area of the $\triangle MNO$, therefore, becomes

$$\mathbf{A}=\frac{1}{8abc\sin A\sin B\sin C}\int_0^a\int_0^b\int_0^c(\mathbf{N})^2dxdydz$$

$$=\tfrac{1}{24}\left(\frac{a^2\sin A}{\sin B\sin C}+\frac{b^2\sin B}{\sin C\sin A}+\frac{c^2\sin C}{\sin A\sin B}\right)=\frac{a^4+b^4+c^4}{48\triangle}.$$

[Note.—Problems twenty-three and twenty-four are identical. This fact was not observed until after they were both printed. ED.]

PROBLEMS.

31. **Proposed by B. F. FINKEL, A. M., Professor of Mathematics and Physics in Drury College, Springfield, Missouri.**

Find the average length of a line drawn at random across the opposite sides of a rectangle whose length is l and breadth b.

32. **Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Pennsylvania.**

Find the average area of the random sector whose vertex is a random point in a given circle.

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MISCELLANEOUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

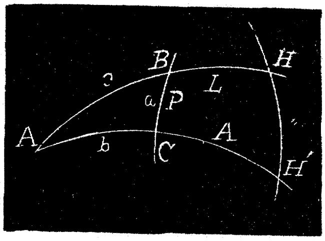
SOLUTIONS OF PROBLEMS.

17. **Proposed by SAMUEL HART WRIGHT, M. D., M. A., Ph. D. Penn Yan, New York.**

A bright star passed my meridian at 7 P. M. The Chronometer soon after ran down and stopped, but I set it again when the same star had a true altitude of $30^\circ=\alpha$. What time was it then, my latitude being $42^\circ\ 30'$ N.$=\lambda$, and the star's declination 60° N.?$=\delta$

II. Solution by the PROPOSER.

Let B be the north pole, A the zenith, C the star, HH' the horizon, AH and AH' each $=90^\circ$, AH' being a meridian, AH a verticle circle, BH' the altitude of the pole $=$ the latitude $=L$, $AB=$co-latitude $=c$, $BC=a=$ polar distance of the star$=P$, $AC=b=$the zenith distance of star, $CH=A=$altitude of the star, and the angle $ABC=$the hour-angle of the star$=T$ in siderial time. Put $s=\frac{1}{2}(a+b+c)$, and $s-a=a'$, $s-b=b'$, and $s-c=c'$. Then by *Sph. Trig.*



$\sin\frac{1}{2}T=\sqrt{\left(\frac{\sin c'\sin a'}{\sin c\sin a}\right)}$, and